Int. J. Heat Mass Transfer. Vol. 10, pp. 745–754. Pergamon Press Ltd. 1967. Printed in Great Britain

CALCULATION OF THE THERMAL CONDUCTIVITY OF NORMAL HYDROGEN IN THE DENSE GASEOUS AND LIQUID STATES†

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(*Received* 22 *August* 1966 *and in revised form* 2 *December* 1966)

Abstract—Thermal conductivity data for saturated liquid, normal hydrogen and for the gas compressed to 500 atm between 78° and 300°K, have been fitted to a simple function relating excess thermal conductivity, density and temperature. The temperature dependence of this function was derived on the basis of a lattice theory of thermal conductivity. It has been used to calculate the thermal conductivity of dense fluid hydrogen between the triple point and 80°K, at pressures up to 350 atm where there are no experimental results.

NOMENCLATURE

C_s,	velocity of sound;
C_v,	molar heat capacity at constant volume;
D,	Debye function;
k,	thermal conductivity;
k^*,	excess thermal conductivity;
k_0,	thermal conductivity at 1 atm;
ℓ,	phonon mean free path;
R,	gas constant;
T,	temperature;
T_t,	triple point temperature.
Greek symbols	
γ,	Grüneisen constant;
θ,	Debye temperature;
θ_0,	Debye temperature at 0°K;
ν,	lattice vibration frequency;
ρ,	density.
Subscript	
σ,	saturated liquid–vapour curve.

INTRODUCTION

THE THERMODYNAMIC properties [1] and viscosity [2] of hydrogen have been investigated over a wide range of temperature and density during recent years. Thermal conductivity results for the dense fluid, however, are restricted to measurements by Johnston *et al.* [3] on the saturated liquid between 15° and 25°K, and very recently by Golubev and Kal'sina [4] on the gas compressed to 500 atm at 78°K and above. In the extensive region between these two sets of measurements the thermal conductivity, k, is very sensitive to temperature and pressure, and is of theoretical interest because hydrogen is a member of the small group of quantum fluids (He^3, He^4, H_2, D_2, T_2 and Ne). The excess thermal conductivities of classical fluids, and of hydrogen above 78°K, are unique, monotonically increasing functions of density. This leads one to expect that the temperature derivative of the thermal conductivity along the saturated liquid curve $(\partial k/\partial T)_\sigma$ will always be negative, as turns out to be the case for all classical liquids. However, Rogers *et al.* [5] have emphasized that $(\partial k/\partial T)_\sigma$ is positive over a

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considerable temperature range for hydrogen and other quantum fluids.

Prior to the recently published measurements down to 78°K [4] there were two attempts to estimate the thermal conductivity of compressed gaseous and liquid hydrogen. In both cases results for the compressed gas were only available above 300°K at densities up to half the critical density, and drastic interpolations were required. Rogers and Brickwedde [6] used the quantum mechanical principle of corresponding states to account for a positive $(\partial k/\partial T)_\sigma$, but were only able to make a qualitative estimate of the thermal conductivity of the compressed liquid and gas. The interpolation of Schaefer and Thodos [7] does not yield a positive $(\partial k/\partial T)_\sigma$.

The thermal conductivity calculations in this paper, however, do make use of the latest data down to 78°K. The author has indicated in a previous paper [8] that a lattice theory of thermal conductivity can predict a positive $(\partial k/\partial T)_\sigma$ at low temperatures, as a result of the positive temperature derivative of the specific heat of a lattice below its Debye temperature θ. A smooth relationship between θ and density is presented in this paper, which is quantitatively consistent with the specific heat of the solid as a function of density, the thermal conductivity of the saturated liquid and the classical behaviour of the thermal conductivity above 78°K. This is used in conjunction with the unique relationship between excess thermal conductivity and density from Golubev and Kal'sina's data in the classical region, to calculate the thermal conductivity of normal hydrogen between the triple point and 80°K at pressures up to 350 atm.

THE EXCESS THERMAL CONDUCTIVITY OF FLUIDS

The thermal conductivity of a gas is independent of pressure [9] for the pressure range in which the mean free path of the molecules is smaller than the dimensions of the container (and measuring probe), but much larger than their own diameter. The very low pressure thermal conductivity is that of the well understood Knudsen gas [10] and will not be discussed.

At high pressures the thermal conductivity increases above the low pressure limit as a result of an additional mechanism, sometimes called collisional transfer, but denoted by vibrational transfer of energy in this paper. Under these conditions the configurational potential energy of the assembly of molecules differs significantly from zero, and the system as a whole acquires vibrational degrees of freedom. Vibrational wave (i.e. phonon) transport can then occur in the presence of a temperature gradient. These vibrations should be distinguished from intramolecular vibrations which occur within the individual diatomic molecules of the fluid. The excess thermal conductivity $k^*(P, T)$ is commonly measured as the difference between the thermal conductivity of the dense gas or liquid, and that of the gas at 1 atm pressure and the same temperature, $k_0(T)$. In a two phase region at temperatures where the vapour pressure is less than 1 atm, $k_0(T)$ is the thermal conductivity of the vapour.

$k^*(P, T)$ is a function of the density, ρ, only and can be written:

$$k^*(P, T) = k^*(\rho) \qquad (1)$$

for many substances over a wide range of conditions excluding the immediate vicinity of the critical point. Equation (1) is the basis of many correlations of the thermal conductivities of simple fluids in the liquid and dense gaseous states [11]. In the critical region there is an anomalous increase in thermal conductivity, which has been thoroughly investigated for CO_2 by Sengers [12] and for NH_3 by Needham and Ziebland [13], but its origin is obscure, and it is not possible to make an *a priori* estimate of the magnitude of the enhancement for hydrogen. However, it will occur only in a very narrow range of (P, T) conditions and is ignored in the present work.

THE THERMAL CONDUCTIVITY OF SOLIDS AND LIQUIDS

Intuitively one might expect the mechanism of vibrational energy transfer to be similar in liquids and solids. The thermal conductivity of solids is fairly well understood qualitatively and quantitatively [14]. Thermal energy is transmitted in vibrational waves and the corresponding particles are phonons. The less rigorous theories assuming with Einstein a single lattice frequency, ν, give:

$$k_{\text{solid}} = \tfrac{1}{3}\rho\, \ell\, C_s C_v \qquad (2)$$

where ρ is the density in molar units, ℓ is the mean free path of the phonon, C_s is the velocity of sound in the solid and C_v is the molar heat capacity at constant volume. ℓ and hence k are limited only by the size of the sample in a perfect harmonic solid. In a real solid the phonons are scattered by:

(i) anharmonic components in the lattice vibrations (Umklapp processes), and
(ii) lattice imperfections including dislocations, vacancies, impurities and other disorders.

Equations similar to (2) have been used to explain the thermal conductivity of glasses [15] and polymers [16] at low temperatures. More rigorous derivations using the Debye spectrum for the solid frequencies have resulted in complex equations. Their solutions have successfully predicted the temperature dependence of the thermal conductivity of crystalline solids at low temperatures [17] where k, increasing with $1/T$ becomes large as the lattice vibrations become increasingly harmonic. However, k always passes through a maximum close to 0°K when lattice defect scattering takes precedence. Then k falls rapidly to zero as a result of the behaviour of C_v close to 0°K.

The disordered structure of the glassy state defines a relatively low limit for ℓ at all times, so that k decreases quite smoothly from the room temperature value to zero at 0°K without the occurrence of a maximum for these materials.

Eyring *et al.* [18] and McLaughlin *et al.* [19] have used lattice theories and expressions like equation (2) to predict the thermal conductivities of classical liquids below their critical points, at densities not much less than the solid density. Under these conditions X-ray evidence suggests that liquids have a disordered lattice structure [20], which emphasizes the similarity between the glassy and liquid states, and indicates that the calculations are not entirely inappropriate. Predictions by Eyring *et al.* of the temperature and pressure dependence of the thermal conductivity of argon, nitrogen and methane agree with experiment to within 4 per cent.

Lattice theories are applied to quantum fluids in this paper, and equation (2) is used to estimate how equation (1) (which gives the excess thermal conductivity of hydrogen as a function of density) must be modified at very low temperatures. The lattice thermal conductivity, of course, is equivalent to the excess thermal conductivity, k^*, of the fluid and not the total conductivity k. k_0 arises from an entirely different mechanism. Hence:

$$k^*(P, T) = \tfrac{1}{3}\rho\, \ell\, C_s C_v \qquad (3)$$

where ρ, ℓ and C_s are the density, phonon mean free path and velocity of sound for the fluid. C_v here corresponds to the "vibrational" molar specific heat of the fluid and must not be confused with the actual measured specific heat, whose non-vibrational components do not contribute to this thermal conductivity mechanism. It may be thought of as the specific heat of the solid extrapolated to the temperature and density of the fluid in the absence of melting. The excess or residual specific heat C_v^*, being the difference between the specific heat of a fluid and of the perfect gas at the same temperature, is much used in the thermodynamics and statistical mechanics of classical fluids [21]. It is less meaningful for hydrogen as it becomes negative for the saturated liquid [1].

An exact adherence to the Debye theory of specific heats [22] will not occur but nevertheless C_v will be written:

$$C_v = 3R\,D[\theta(\rho)/T]. \tag{4}$$

R is the gas constant, θ the Debye temperature and $D[\theta(\rho)/T]$ the Debye function, which goes to zero with T^3 at 0°K, and approaches unity asymptotically at temperatures above θ.

Table 1. The triple point temperatures and Debye temperatures for hydrogen, argon and krypton

Substance	Triple point temperature (T_t °K)	Debye temperature (θ°K)
Hydrogen	14	100
Argon	84	85
Krypton	116	65

Table 1 contains the triple point temperatures T_t, and approximate Debye temperatures θ for the solid at zero pressure, for hydrogen [23], argon [24] and krypton [24]. The relative magnitudes of T_t and θ indicate that hydrogen can exist as a dense fluid at temperatures which are below θ, as well as at temperatures above θ. Krypton can only exist as a dense fluid above θ and argon is a borderline case.

Thus it becomes clear that equation (3) has all the required characteristics. It can account for classical fluids (T always greater than θ) having only one form, equation (5), for k^* throughout the dense gaseous and liquid regions:

$$k^*(P, T) = \rho\,\ell\,C_s R = k^*(\rho). \tag{5}$$

It can also account for k^* for hydrogen, following the same form, equation (5), at high temperatures and another form, equation (6), at low temperatures.

$$k^*(P, T) = \rho\,\ell\,C_s R D[\theta(\rho)/T]. \tag{6}$$

Equation (5) states that the product $\rho\,\ell\,C_s R$ (and therefore $\ell\,C_S$) is determined unambiguously by density in the classical region, and we assume here that its value is the same in the quantum region at the same density for the same substance. Eliminating it from equations (5) and (6) one obtains equation (7).

$$k^*(P, T) = k^*(\rho)\,D[\theta(\rho)/T]. \tag{7}$$

It is clear that a positive $(\partial k^*/\partial T)_\sigma$ can arise from the positive temperature derivative of $D[\theta(\rho)/T]$.

In the next section the expressions $k^*(\rho)$ and $\theta(\rho)$ are obtained as functions of density, from experimental data on hydrogen, and equation (7) is then used to calculate the thermal conductivity of gaseous and liquid hydrogen in regions where density measurements are available.

THE THERMAL CONDUCTIVITY OF HYDROGEN

The thermal conductivity of hydrogen has been measured recently at temperatures between −195° and 20°C, at pressures up to 500 atm [4]. The authors, Golubev and Kal'sina, do not specify the ortho-para composition of their hydrogen, but since they do not mention a catalytic converter, the measurements were probably made on normal hydrogen. Confirmation of this assumption is found in close agreement between their 1 atm data and the accepted values [25] for normal hydrogen.

The maximum density of the compressed gas in these measurements 0·036 mol cm^{-3} is as large as the density of the saturated liquid. Golubev and Kal'sina found that the excess thermal conductivity was a unique function of density over the entire temperature range, and they use this relationship to smooth their data, which they present at integral temperature and pressure intervals. However, they do not present their $k^*(\rho)$ relationship, nor do they specify the source of their density data. This relationship was reproduced using their $k^*(P, T)$ data and the $\rho(P, T)$ data of Michels *et al.* [26]. Experimental density results are not available for the highest pressures and lowest temperatures of the thermal conductivity data. These densities were obtained by linear extrapolation of P versus T isochores from Michel's data for normal hydrogen, and N.B.S. data for parahydrogen [27]. The use of density data for both forms of hydrogen in this extrapolation does not introduce significant error, because the density is almost independent of ortho-para composition [1].

$k^*(\rho)/\rho$ is presented graphically in Fig. 1 for temperatures between −195° and −150°C (78° and 123°K). Most points lie within two per cent of the smooth curve. Smoothed values of $k^*(\rho)$ are presented in Table 2.

At densities below $2{\cdot}5 \times 10^{-2}$ mol cm^{-3}, k^* is well represented by a quadratic equation in ρ:

$$k^*(\rho) = 0{\cdot}308\,\rho + 0{\cdot}1644\,\rho^2, \qquad (8)$$

$k^*(\rho)$ is the residual thermal conductivity $\times 10^4$ in cal cm^{-1} s^{-1} degK^{-1}, and ρ is the density $\times 10^2$ in mol cm^{-3}.

$\theta(\rho)$ as a function of ρ was obtained from Johnston's data for the thermal conductivity of the saturated liquid. Each of his measured values of $k(P, T)_\sigma$ was manipulated as follows:

(i) subtraction of the appropriate $k_0(T)$ [25] from $k(P, T)_\sigma$ to give $k^*(P, T)_\sigma$;

(ii) specification of ρ at the temperature (and pressure) of the measurement [1];

(iii) determination of $k^*(\rho)$ from the tabulated values of $k^*(\rho)$ versus ρ in Table 2;

(iv) calculation of $D[\theta(\rho)/T]$ using equation (7), and

(v) calculation of $\theta(\rho)/T$ and then $\theta(\rho)$ using tabulation of $D(\theta/T)$ and the temperature of Johnston's $k(P, T)$ measurement.

Table 2. Excess thermal conductivity for compressed normal hydrogen gas as a function of density above 78°K

Density ($\rho \times 10^2$ mol cm^{-3})	Residual thermal conductivity ($k^* \times 10^4$ cal cm^{-1} s^{-1} degK^{-1})	Density ($\rho \times 10^2$ mol cm^{-3})	Residual thermal conductivity ($k^* \times 10^4$ cal cm^{-1} s^{-1} degK^{-1})
0·0	0·000	2·0	1·272
0·2	0·068	2·2	1·478
0·4	0·150	2·4	1·690
0·6	0·244	2·6	1·895
0·8	0·352	2·8	2·116
1·0	0·473	3·0	2·376
1·2	0·607	3·2	2·656
1·4	0·756	3·4	2·999
1·6	0·915	3·6	3·424
1·8	1·087	3·8	3·914
		4·0	4·520
		4·2	5·237
		4·4	6·072

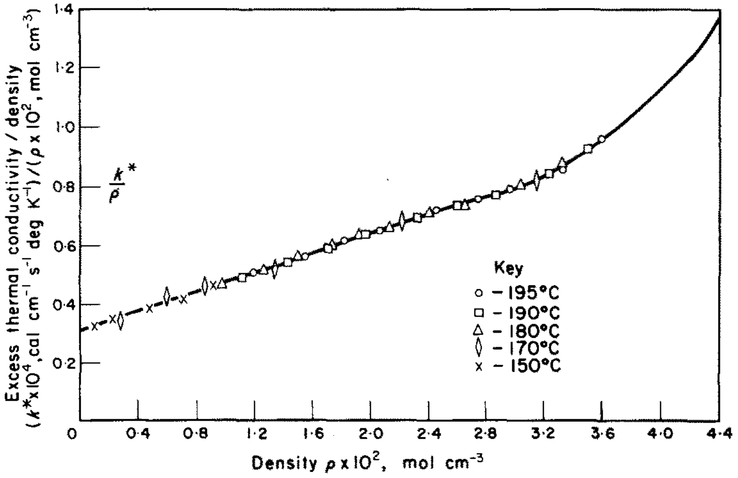

FIG. 1. The excess thermal conductivity of compressed gaseous hydrogen above 78°K.

The results are plotted in Fig. 2. Ahlers [23] has determined $\theta(\rho)$ at higher densities from his measurements of C_v for solid hydrogen as a function of temperature and pressure. The Debye theory of lattice vibrations does not allow for any temperature variation of θ. Nevertheless the Debye θ's of argon [24], krypton [24] and hydrogen [23] decrease by about 10 per cent from θ_0 at 0°K and pass through shallow minima at about 15°K. Non-vibrational contributions to C_v due to vacancy formation complicate the interpretation at higher temperatures, and it is assumed in this paper that the 15°K values are appropriate at all higher temperatures. These are included in Fig. 2 at the three densities of the measurements on solid hydrogen. The data for the solid and liquid are well fitted by a linear function of ρ:

$$\theta(\rho) = 67 \times 10^2 \rho - 189{\cdot}5 \tag{9}$$

where the units of ρ are mol cm^{-3} and of θ, °K. Clearly this expression predicts negative θ's at densities below $2{\cdot}82 \times 10^{-2}$ mol cm^{-3} so that it is not used below this density in the following calculations. Presumably $\theta(\rho)$ curves sharply in this region and approaches zero asymptotically at $\rho = 0$. However, the exact form of $\theta(\rho)$ is insignificant here because T/θ is always large, and $D(\theta/T)$ equal to unity. The expression:

$$\theta(\rho_1) = \theta(\rho_2)(\rho_1/\rho_2)^{\gamma} \tag{10}$$

where γ is the Grüneisen constant, fits the solid data fairly well [23], but a plot of log θ versus ρ through the liquid and solid data is strongly curved, indicating a variable γ, and was rejected in favour of equation (9) for interpolation of $\theta(\rho)$ data.

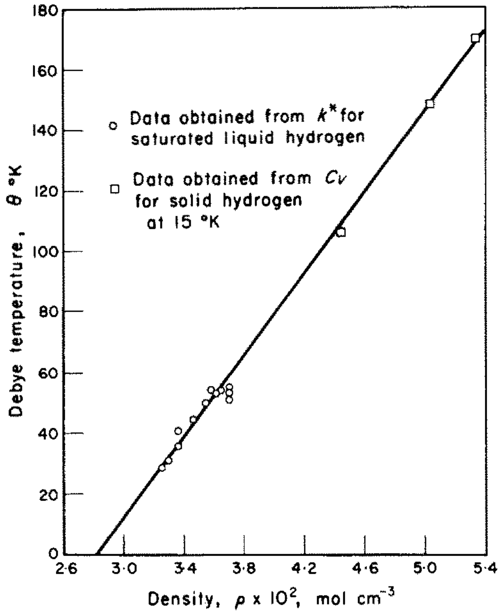

FIG. 2. Debye temperatures for hydrogen obtained from the thermal conductivity of the saturated liquid and the specific heat of the solid.

The thermal conductivity of the dense fluid was then calculated at integral temperatures and pressures using the following procedure:

(i) evaluation of the appropriate density from N.B.S. tabulations [27] (the density of normal hydrogen is only marginally different from that of parahydrogen [1], and the difference may be ignored for these calculations);
(ii) evaluation of the classical value of $k^*(\rho)$ using Table 2, and determination of $\theta(\rho)$ and hence $T/\theta(\rho)$ using equation (9) and the appropriate temperature;
(iii) determination of $k^*(P, T)$ using equation (7) and tabulations of the Debye function;
(iv) Addition of $k^*(P, T)$ and the appropriate $k_0(T)$ for normal hydrogen [25] to give $k(P, T)$.

RESULTS AND DISCUSSION

The thermal conductivity of dense fluid normal hydrogen, calculated as described in the previous section, is presented in Table 3 at integral pressures (up to 350 atm) and temperatures between 17° and 80°K. The thermal conductivity of the saturated liquid and vapour, and of the freezing liquid is given at close temperature intervals in Table 4. Thermal

Table 3. Thermal conductivity of normal hydrogen at integral pressures and temperatures $k + 10^4$, cal cm^{-1} s^{-1} $degK^{-1}$

Temp (°K) \ P (atm)	0	5	10	15	20	30	50	75	100	150	200	250	300	350
17	0·31	2·46	2·44	2·42	2·39	2·38	2·28	2·20	2·14					
18	0·33	2·62	2·63	2·60	2·59	2·54	2·48	2·40	2·35					
20	0·37	2·89	2·84	2·86	2·87	2·83	2·79	2·76	2·71	2·63	2·56			
22	0·40	2·99	3·02	3·03	3·02	3·03	3·04	3·02	3·00	2·95	2·91	2·86	2·82	
24	0·44	3·02	3·05	3·08	3·10	3·14	3·17	3·22	3·24	3·23	3·23	3·21	3·16	3·14
25	0·46	2·99	3·04	3·08	3·10	3·17	3·23	3·28	3·32	3·33	3·35	3·37	3·33	3·33
26	0·48	2·96	3·00	3·05	3·09	3·16	3·26	3·32	3·38	3·42	3·47	3·49	3·51	3·48
28	0·52	0·62	2·88	2·95	3·02	3·11	3·26	3·39	3·48	3·60	3·67	3·72	3·76	3·76
30	0·55	0·64	0·64	2·76	2·88	3·02	3·22	3·39	3·52	3·67	3·81	3·89	3·97	4·02
32	0·58	0·66	0·84	2·45	2·64	2·87	3·13	3·35	3·50	3·73	3·87	4·03	4·14	4·20
33	0·60	0·67	0·83	2·23	2·50	2·77	3·08	3·32	3·49	3·75	3·99	4·07	4·19	4·31
34	0·61	0·68	0·81	1·67	2·32	2·66	3·02	3·27	3·46	3·74	3·94	4·11	4·25	4·36
35	0·63	0·70	0·81	1·14	2·09	2·56	2·96	3·23	3·44	3·74	3·96	4·14	4·30	4·41
36	0·65	0·71	0·82	1·05	1·78	2·44	2·89	3·18	3·41	3·74	3·97	4·16	4·33	4·47
38	0·68	0·74	0·83	0·99	1·31	2·15	2·73	3·10	3·32	3·69	3·96	4·17	4·38	4·55
40	0·71	0·77	0·85	0·97	1·17	1·84	2·57	2·98	3·24	3·63	3·94	4·18	4·42	4·59
50	0·87	0·91	0·96	1·03	1·10	1·31	1·86	2·43	2·80	3·30	3·66	3·99	4·28	4·54
60	1·01	1·04	1·08	1·13	1·18	1·30	1·62	2·05	2·43	2·97	3·37	3·69	3·99	4·30
70	1·15	1·18	1·21	1·25	1·29	1·37	1·59	1·90	2·21	2·75	3·14	3·47	3·75	4·03
80	1·31	1·33	1·36	1·39	1·42	1·49	1·66	1·89	2·14	2·61	3·01	3·32	3·60	3·86

conductivity isobars up to 350 atm from the triple point to 80°K, and the freezing liquid and saturated liquid and vapour data are presented in Fig. 3.

It is clear that the calculated curves closely fit the experimental data for the saturated liquid, and for the compressed gas at 78°K. Calculations for the saturated liquid and vapour were continued up to the critical point and in Fig. 3 are shown to run into each other smoothly at $k = 1{\cdot}43 \times 10^{-4}$ cal cm^{-1} s^{-1} $degK^{-1}$. However, work referred to earlier [12, 13] indicates that molecular clustering will cause an anomalous increase in k in this region. Clustering occurs in the (P, T) region where the compressibility becomes very large near to, and infinite at, the critical point. It is expected that the calculated data will be unreliable here and is indicated by broken curves in Fig. 3.

The calculated values of k should be reliable to a few per cent in the remainder of the (P, T) range.

$D[\theta(\rho)/T]$ assumes the value of 0·998 at 80°K and 350 atm, and 0·98 at 78°K and 500 atm, which is the lowest temperature and highest density of Golubev and Kal'sina's measurements. Hence these calculations are fully consistent with the classical behaviour of the thermal conductivity of hydrogen at 78°K and above. At the critical density classical behaviour is maintained at all temperatures down to the critical temperature.

A comparison with previous estimates is of interest. The correlation of Rogers *et al.* [6] indicates that compression of the saturated liquid will cause a reversion to classical behaviour. This paper, however, predicts that compression will bring about even larger deviations from classical values, as a result of the increased value of θ at higher densities. At 78·15°K, the predictions of Schaefer and Thodos [7] agree with the measurements of Golubev and Kal'sina (and this work) at 150 atm only. They are too large by 13 per cent at 50 atm and

Table 4. Thermal conductivity of saturated liquid and vapour and freezing liquid normal hydrogen

Temp. (°K)	Density [1] ($\rho \times 10^2$ mol cm^{-3})			Thermal conductivity ($k \times 10^4$ cal cm^{-1} s^{-1} degK^{-1})		
	Saturated liquid	Saturated vapour	Freezing liquid	Saturated liquid	Saturated vapour	Freezing liquid
14	3·83	0·006	3·83	1·83	0·25	1·83
15	3·79	0·010	3·92	2·05	0·27	1·92
16	3·74	0·016	4·00	2·29	0·29	2·01
17	3·70	0·023	4·08	2·48	0·32	2·15
18	3·65	0·033	4·15	2·62	0·34	2·24
19	3·60	0·045	4·22	2·76	0·36	2·37
20	3·54	0·060	4·29	2·89	0·38	2·49
21	3·48	0·078	4·35	2·97	0·41	2·65
22	3·42	0·100	4·41	2·99	0·43	2·82
23	3·36	0·126	4·47	3·02	0·46	3·00
24	3·29	0·157		3·01	0·49	
25	3·21	0·194		2·99	0·52	
26	3·13	0·238		2·94	0·56	
27	3·04	0·290		2·88	0·60	
28	2·94	0·353		2·79	0·64	
29	2·82	0·429		2·67	0·69	
30	2·69	0·524		2·54	0·75	
31	2·53	0·648		2·39	0·83	
32	2·31			2·18		
33	1·90			1·78		
33·18	1·49	1·49		1·43	1·43	

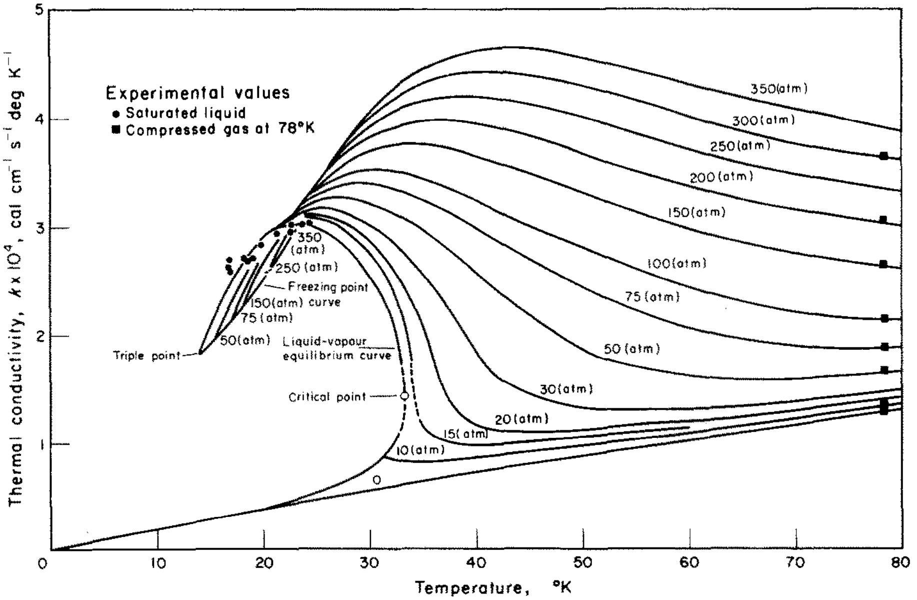

FIG. 3. Thermal conductivity isobars for normal hydrogen from the triple point to 80°K at pressures up to 350 atm.

too small by 9 per cent at 300 atm. At the critical temperature above 50 atm they are smaller by 15–18 per cent than the values in this paper. Their predictions for the saturated and compressed liquid are quite different from these because of their neglect of the positive value of $(\partial k/\partial T)_\sigma$. At 20°K they predict that a pressure of 100 atm increases the thermal conductivity of the liquid by 12 per cent, whereas this paper predicts a decrease of 6 per cent.

Values of $k^*(\rho)$ at densities up to $1{\cdot}8 \times 10^{-2}$ mol cm^{-3} from Hamrin and Thodos's [28] very recent measurements above 0°C, are about 30 per cent higher than those used in this paper. However, since their individual measurements of $k^*(\rho)$ are scattered by about ± 15 per cent, and the results apply to a much higher temperature range, this discrepancy is not significant to the results of this paper. k^* is less than one quarter of k above 0°C, even at 660 atm—the highest pressure of Hamrin and Thodos's measurement, which accounts for the wide spread in their k^* data.

ACKNOWLEDGEMENTS

The author is indebted to Mr. H. Ziebland for helpful advice and encouragement.

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Résumé—Les résultats de conductivité termique pour l'hydrogène normal liquide saturé et pour le gaz comprimé à 500 atm entre 78° et 300°K, ont été corrélés par une fonction simple reliant la conductivité thermique supplémentaire, la masse volumique et la température.

La dépendance de cette fonction par rapport à la température à été obtenue sur la base d'une théorie de la conductivité thermique avec le modèle du réseau. On l'a employé pour calculer la conductivité thermique de l'hydrogène fluide dense entre le point triple et 80°K, à des pressions allant jusqu'à 350 atm pour lesquelles il n'y a pas de résultats expérimentaux.

Zusammenfassung—Werte der Wärmeleitfähigkeit für gesättigte Flüssigkeit, Normalwasserstoff und für ein Gas, das auf 500 atm komprimiert wurde im Bereich 78–300°K, wurden einer einfachen Funktion angepasst, die eine Beziehung herstellt zwischen Erhöhung der Wärmeleitfähigkeit, Dichte und Temperatur. Sie Temperaturabhängigkeit dieser Funktion wurde mit Hilfe einer Gittertheorie der Wärmeleitfähigkeit von Wasserstoff zwischen Tripelpunkt und 80°K, bei Drücken bis zu 350 atm. Dafür lagen bisher keine Versuchsergebnisse vor.

Аннотация—Теплопроводность насыщенного жидкого обычного водорода и газа, сжатого до 500 атм при 78–300°K, описывается простой функцией, учитывающей избыточную теплопроводность, плотность и температуру. На основании решеточной теории теплопроводности получена температурная зависимость для этой функции. Она использовалась для расчетов теплопроводности плотного водорога при температуре от тройной точки до 80°K и давлении до 350 атм, экспериментальные данные для которых отсутствуют.